

# Non-Classical Methods for the Simulation and Design of Quantum Materials

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April 2026

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# Introduction and Motivation

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# Motivation: Materials Science Perspective

- Materials Science has advanced in various **paradigms** [AC16].
- Each paradigm is defined by the **novel tools and theories** developed to understand new physical phenomena:

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A. Agrawal and A. Choudhary

APL Mater. 4, 053208 (2016)

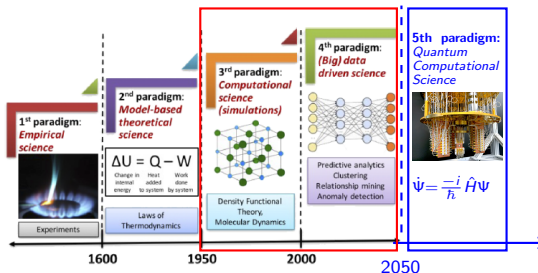
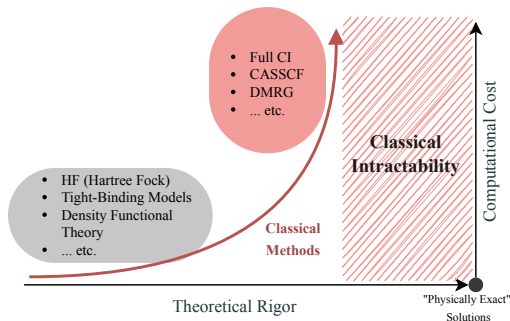


FIG. 1. The four paradigms of science: empirical, theoretical, computational, and data-driven.

- However, due to the failure of classical computational methods to model quantum materials at scale, **new tools are required**.

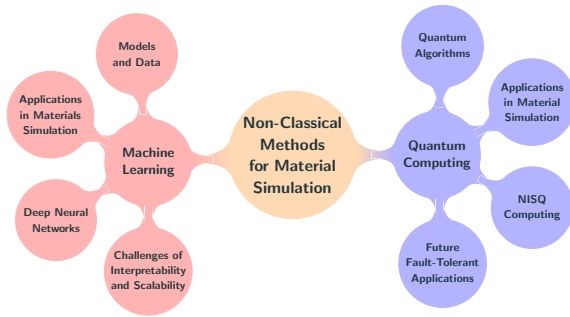
# Classical Effective Models

- Finding an exact solution to the electronic ground state is intractable on a classical computer (requires diagonalizing  $\hat{H}$ ).
- One common solution is to find an effective  $\hat{H} = \sum_{i,j} h_{ij} \hat{c}_i^\dagger \hat{c}_j$  that approximately accounts for second-order (Coulomb) terms.
- Classical computational methods can accomplish this at various degrees of physical rigor and scale, **but they can be quite costly**:



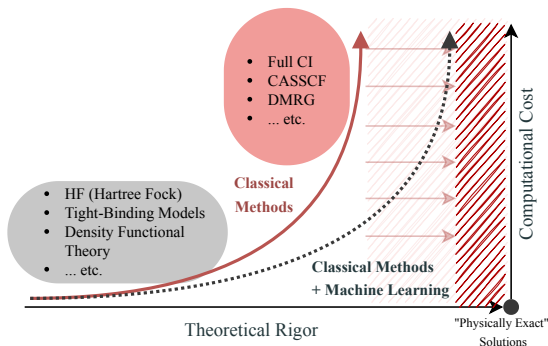
# Non-Classical Methods

- My research studies alternatives to classical methods, which I refer to here as **non-classical methods**.
- Specifically, I focus on how emerging computational techniques and technologies like **machine learning** and **quantum computing** can be applied to solve problems beyond the reach of classical methods:



# Machine Learning

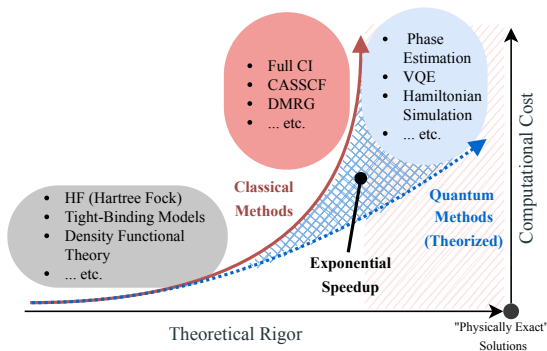
- Augmenting classical methods with machine learning surrogates has shown a strong ability to reduce computational cost.
- This enables new kinds of problems to be solved which were previously considered to be intractable:



- Achieving this speedup requires **lots of reliable training data**.

# Quantum Advantage in Chemistry

- The electronic structure problem can be solved on a quantum computer using Phase Estimation and VQE (Variational Quantum Eigensolver) methods [PM19, BGMT17, ESL<sup>+</sup>20].
- For Hamiltonians with local interactions, quantum computing methods can give up to an **exponential speedup** for certain systems [CST<sup>+</sup>21, DBK<sup>+</sup>22]:



# My Dissertation Research



# Overview of Projects

Here, I will focus on three projects, highlighting my recent applications of non-classical methods for materials simulation:

1. High-throughput screening study of superconductors
2. Simulating open quantum systems on quantum computers
3. Quantum-informed neural networks (QuINNs) for electronic structure modeling

If time, we can also discuss future directions of these projects and my ongoing work.

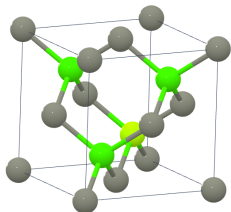
# High-Throughput Screening of Superconductors with Machine Learning

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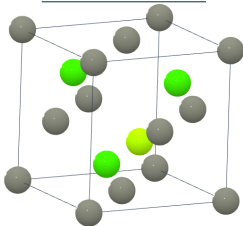
# Graph Neural Networks for Material Property Prediction

- Predicting material superconductivity can be viewed as an atomic structure-property learning problem.
- Atomic structures are naturally interpreted as periodic graphs (networks of nodes connected by edges):

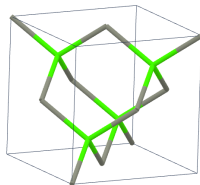
Crystal (graph)



Atoms (nodes)



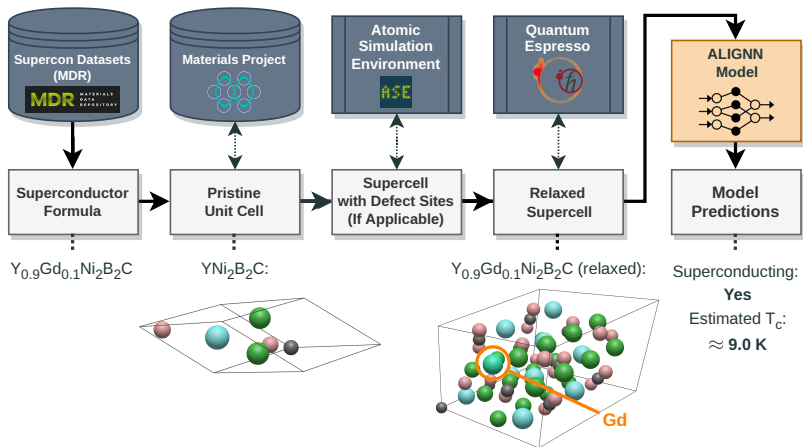
Bonds (edges)



- **Graph neural networks** are neural network models that make predictions from labeled graphs.
- Crystal structure graphs contain node labels (e.g., atomic species), and edge labels (e.g., bond lengths and bond angles).

# Superconductor Dataset Pipeline

## Data Generation Pipeline



# Identified Superconductors

- We trained ALIGNN to classify superconductors (Acc:  $\sim 90\%$ , AUROC: 0.74) and estimate  $T_c$  (Err:  $\pm 2.2K$ ), and then used the model to screen over 40,000 novel metals, oxides, and hydrides from the Materials Project database [JOH<sup>+</sup>13].
- We identified **over 600 candidate superconductors** not contained in our dataset:

| Materials Project ID | Formula                                       | Stable | Experimentally Observed | Predicted Mean $T_c$ | Closest Known Superconductor |
|----------------------|---|--------|-------------------------|----------------------|------------------------------|
| mp-672238            | CeCuSb <sub>2</sub>                           | Yes    | Yes                     | 1.88 K               | Cu <sub>2</sub> Sb           |
| mp-1025564           | LuAl <sub>2</sub> Pd <sub>5</sub>             | Yes    | No                      | 8.12 K               | Pd                           |
| mp-10898             | ScAlNi <sub>2</sub>                           | Yes    | Yes                     | 1.25 K               | Ni <sub>3</sub> Al           |
| mp-573601            | Th <sub>7</sub> Ru <sub>3</sub>               | Yes    | Yes                     | 0.95 K               | Th                           |
| mp-28280             | K <sub>5</sub> V <sub>3</sub> O <sub>10</sub> | Yes    | Yes                     | 4.93 K               | V                            |
| mp-1224184           | HfZrB <sub>4</sub>                            | Yes    | No                      | 2.57 K               | HfB <sub>2</sub>             |
| mp-1228895           | AlGaSb <sub>2</sub>                           | No     | No                      | 4.15 K               | AlSb                         |
| mp-1222266           | Lu <sub>3</sub> S <sub>4</sub>                | Yes    | No                      | 3.63 K               | LuS                          |
| mp-1079796           | Ti <sub>3</sub> Pd                            | Yes    | No                      | 4.24 K               | Ti                           |
| mp-1218331           | Sr <sub>3</sub> CaSi <sub>8</sub>             | No     | No                      | 1.88 K               | Sr(Si) <sub>2</sub>          |
| mp-1021328           | H <sub>4</sub> C                              | Yes    | No                      | 50.97 K              | H <sub>2</sub>               |
| mp-11494             | LuPb <sub>3</sub>                             | No     | Yes                     | 3.80 K               | Pb                           |
| mp-1226890           | Ce <sub>4</sub> H <sub>11</sub>               | Yes    | No                      | 331.83 K             | CeHg                         |
| mp-22266             | GdB <sub>6</sub>                              | Yes    | Yes                     | 2.22 K               | B                            |
| mp-1184695           | Ho <sub>3</sub> Er                            | No     | No                      | 6.22 K               | Ho                           |
| ...                  | ...   | ...    | ...                     | ...                  | ...                          |

# Simulation of Open Quantum Systems on Quantum Computers

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# Open Quantum Systems

- Understanding the environmental conditions (e.g., high pressure, low temperature, low disorder) that facilitate quantum behavior is crucial to engineering quantum materials [BP02].
- This requires treating a material as an **open quantum system**.
- We represent open systems by a density matrix  $\rho = \sum_{n=1}^N p_n \Psi_n \Psi_n^\dagger$ . ( $n \times n$  for non-interacting electrons,  $2^N \times 2^N$  for interacting).
- Open system dynamics are modeled by the Lindblad equation:

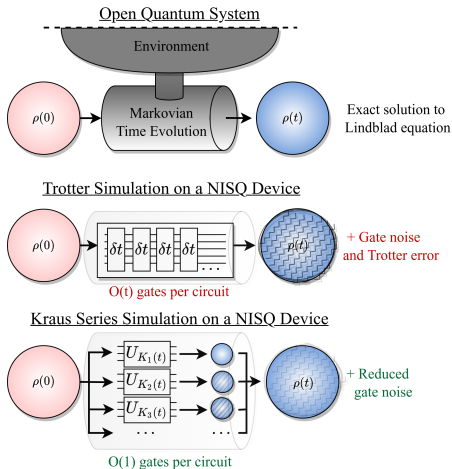
## Lindblad Equation

- $\hat{L}_i$ : Lindblad operators (environment interaction modes)
- $\gamma_i$ : Environment coupling coefficients (decay rates)

$$\frac{\partial}{\partial t} \rho = \underbrace{\frac{-i}{\hbar} [\hat{H}, \rho]}_{\text{Schrödinger (reversible)}} + \underbrace{\sum_i \gamma_i \left( \hat{L}_i \rho \hat{L}_i^\dagger - \frac{1}{2} \{ \hat{L}_i^\dagger \hat{L}_i, \rho \} \right)}_{\text{Dissipative evolution (non-reversible)}} \quad (1)$$

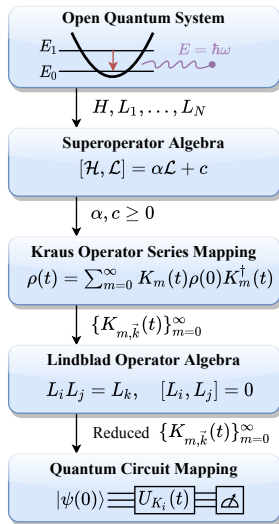
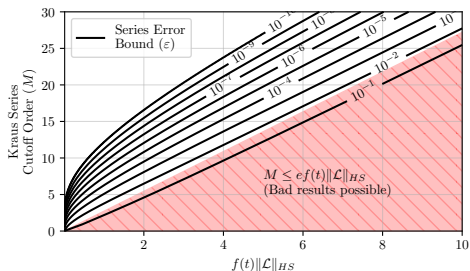
# Solving the Lindblad Equation on a Quantum Computer

- Existing quantum computing methods employ a numerical time-stepped method called **Trotterization**. [Tro59]
- This method is not ideal for NISQ devices, because it produces deep circuits.
- We developed a new simulation method that employs **Kraus series representations**.
- This distributes the computation over many short quantum circuits, reducing device noise.



# The Time-Perturbative Kraus Series Method

- Our method expands the evolution of  $\rho$  as a time-perturbative Kraus operator series:  $\rho(t) = \sum_{m=0}^{\infty} K_m(t)\rho(0)K_m(t)^\dagger$
- Each  $K_m(t)$  (up to a cutoff order  $M$ ) is mapped to a  $t$ -parameterized quantum circuit of bounded depth.
- We proved **asymptotically linear scaling** of  $M$  with evolution time  $t$ :

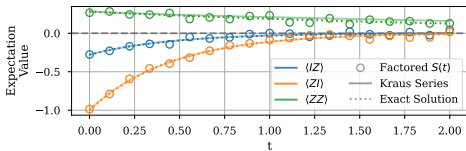
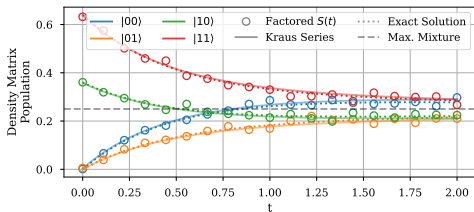


# Simulation of Pauli Channels

- A **Pauli Channel** models decoherence in the qubits on a quantum processor or spin states in magnetic materials.
- For this system, we can even apply all Kraus operators in a superposition  $S(t)$ , yielding an **exponential quantum advantage** over naive classical simulation.

## Two-qubit Crosstalk Channel

- $H, L_1, L_2, L_3, L_4 = 0, IX, XI, ZZ, XX$
- $\gamma_1, \gamma_2 = 1.0$  (Qubit bit-flip errors)
- $\gamma_3, \gamma_4 = 0.1$  (Cross-talk interactions)

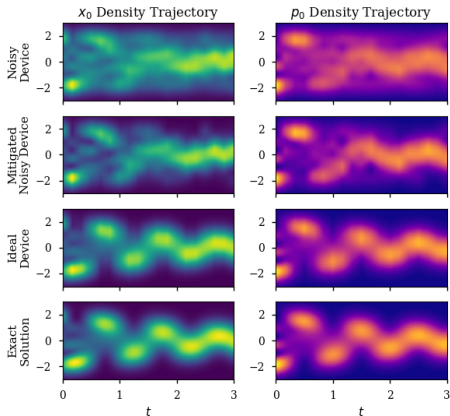


# Simulation of Bosonic Systems & Harmonic Oscillators

- A single mode containing indistinguishable Bosons with energy  $E = \hbar\omega$  can be modeled by a **quantum harmonic oscillator**.
- We model particle loss by applying a damping operator  $L_1 = \hat{a}$  to the oscillator.
- When simulating on NISQ devices, noise must be suppressed through **error mitigation**.

## Damped Harmonic Oscillator

- $H = \hbar\omega(\hat{a}^\dagger a + 1/2) = \hbar\omega(\hat{p}_0^2 + \hat{x}_0^2)/2$
- $L_1 = \hat{a} = (\hat{x}_0 + i\hat{p}_0)/\sqrt{2}, \quad \gamma_1 = 1.0$

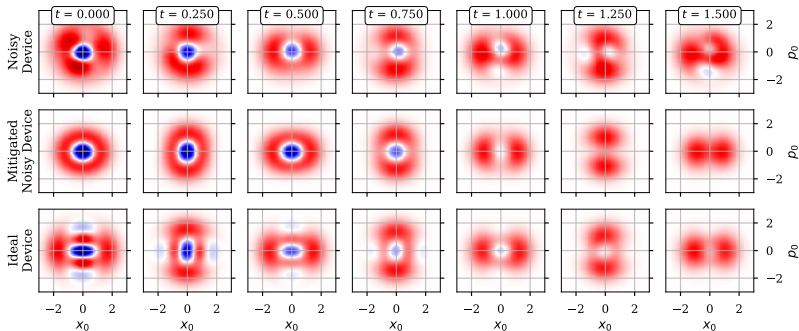


(Results obtained on the IonQ Harmony Quantum Device, June 2024)

# Simulating Phase-Space Dynamics

- Quantum Bosonic systems exhibit behavior that is radically different from classical field-mediated interactions.
- This is observed by simulating the **Wigner Quasi-probability distribution**.

## Simulated Wigner Distribution of a “Schrodinger’s Cat” State



# Quantum-Informed Neural Networks for Electronic Structure Modeling

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# Mean-Field Material Hamiltonians

- Consider an electronic Hamiltonian for periodic material lattices:

$$\hat{H} = \underbrace{\sum_{i,j \in (\text{lattice})}^N h_{ij} \hat{c}_i^\dagger \hat{c}_j}_{N \times N \text{ Matrix Representation}} + \sum_{i,j,k,l=1}^N v_{ijkl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l \quad (2)$$

$2^N \times 2^N$  Matrix Representation

(Here  $i = (n_i, \vec{r}_i)$ , where  $n_i$  is an orbital index and  $\vec{r}_i$  is a site position)

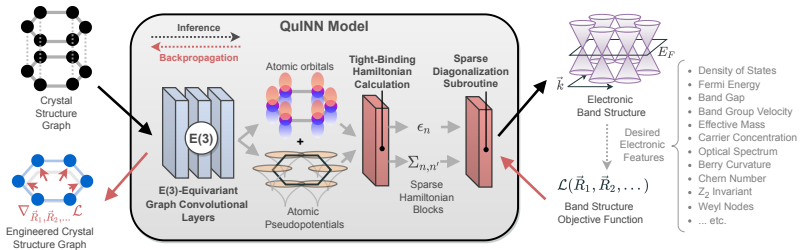
- For electrons that are tightly bound to the lattice sites, we derive an effective mean-field Hamiltonian via the **tight-binding approximation**:

$$\hat{H}(\vec{k}) = \sum_{i,j \in (\text{cell})} e^{i\vec{k} \cdot (\vec{r}_i - \vec{r}_j)} \tilde{h}_{ij} \hat{c}_i^\dagger \hat{c}_j \quad (3)$$

- Above  $\vec{k}$  is a crystal momentum vector in the reciprocal lattice cell.

# QuINN: Quantum-Informed Neural Network Models

- To predict electronic band structure, we have developed a new graph neural network architecture (QuINNs):

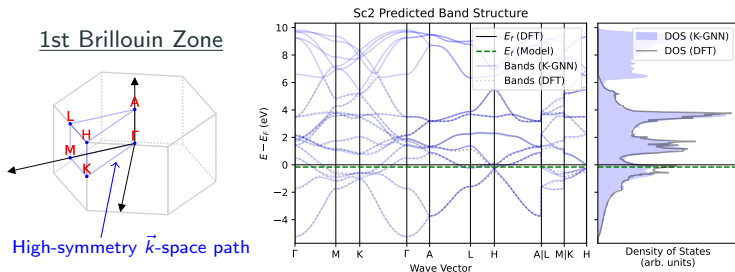


- Since QuINNs are **informed by quantum mechanics**, they can be used to reverse-engineer effective material Hamiltonians and atomic orbitals from electronic structure data.
- This architecture makes **more interpretable predictions** than existing models (like ALIGNN), which are not quantum-informed.

# Predicting Band Structure and Density of States

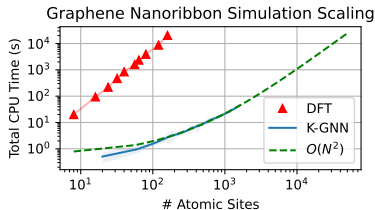
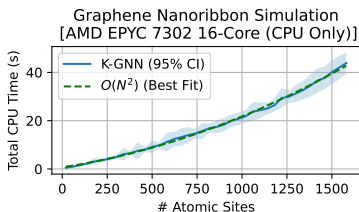
- QuINN models allow for high-resolution sampling of  $\vec{k}$ -space at minimal computational expense (needed for density of states, Fermi surfaces, Green's functions, etc.).
- Computing QuINN bands for each  $\vec{k}$ -point requires a single sparse diagonalization ( $\approx O(N^2)$ ) via the Lanczos algorithm).
- DFT requires many diagonalization steps, each  $O(N^3)$ .

## Example: HCP Scandium, K-GNN (QuINN prototype model)



# QuINN Model Efficiency

- Basic QuINN models compiled through our QuINN Python package's PyTorch backend have demonstrated  $O(N^2)$  scaling.



- Preliminary results suggest that during CPU inference, **10× to 1000×** speedups can be achieved over plane wave DFT methods.
- QuINNs also generalize reasonably well to unseen structures:

|                          | Sc<br>(3D, stable) | Lu<br>(3D, stable) | Dy<br>(3D, stable) | C<br>(2D & 3D, strained) | hBN<br>(2D, strained) |
|--------------------------|--------------------|--------------------|--------------------|--------------------------|-----------------------|
| Validation Set Band RMSE | <b>0.069 eV</b>    | <b>0.122 eV</b>    | <b>0.738 eV</b>    | <b>0.281 eV</b>          | <b>0.195 eV</b>       |

## Future and Ongoing Work **[Skip]**

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# Future and Ongoing Work (Quantum Computing)

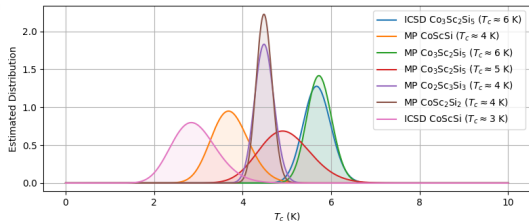
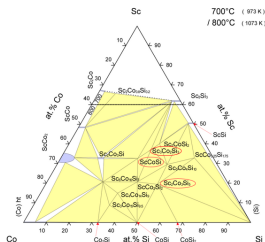
- We have demonstrated that our algorithm allows for accurate simulation of open quantum systems on NISQ hardware.



- Future directions to explore include:
  - Characterizing the kinds of systems where our method yields an exponential quantum advantage.
  - Exploring error mitigation/correction techniques to improve simulation fidelity.
  - Apply Kraus series methods for more efficient subsystem embeddings within VQE (Variational Quantum Eigensolver) methods.

# Screening of Superconducting Materials (Ongoing Work)

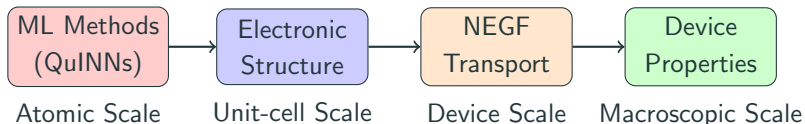
- We have collaborated with Dr. Julia Chan's Lab (Baylor) to attempt synthesis of the most promising intermetallic candidates (Top 10%).
- With further literature review, they confirmed two of the model's top candidate predictions:
  1.  $\text{La}_2\text{Sn}_3$  (Observed  $T_c$ : 2.5 K, Predicted: 2.8 K)
  2.  $\text{Lu}_3\text{Ir}_4\text{Ge}_{13}$  (Observed  $T_c$ : 2.8 K, Predicted: 1.8 K)
- However, many of the top candidates proved difficult to synthesize:
  - Ce-Bi (oxidizes very rapidly, Exhibits Antiferromag. Kondo Effect)
  - Ce-Te (flammable and toxic, Exhibits Charge Density Waves)
  - Sc-Co-Si (difficulty with synthesis and phase purity)



# Future and Ongoing Work (Machine learning)

## 1. Nanoscale Electronic Device Modeling

- QuINNs can be integrated with with **Non-Equilibrium Green's Function (NEGF)** transport models.
- NEGF Theory is necessary to bridge atomic-scale and macroscopic-scale electronic simulations of quantum devices:

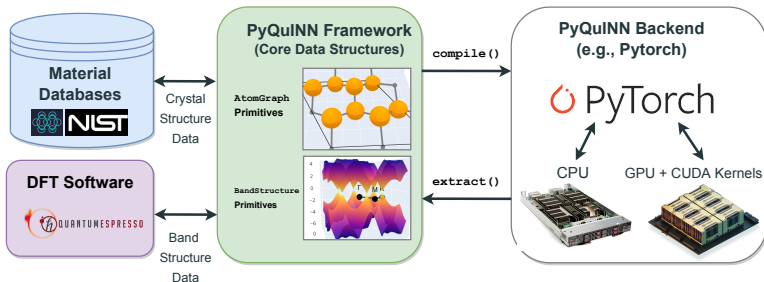


## 2. Hybrid Classical–Quantum Simulation

- QuINNs can compress Hamiltonians for resource-efficient quantum algorithms (**reduced qubit counts and circuit depth for VQE**).
- The PyQuINN framework can also compute differentiable 2nd-quantized Hamiltonians for hybrid variational optimization.

# The PyQuINN Framework

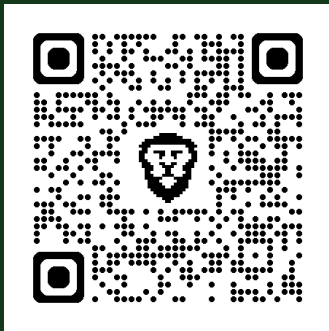
- We have written a framework for QuINN models entirely using Pytorch operations, allowing for efficient CPU inference and GPU training:



- We are working toward packaging and distributing this framework as open-source software on the Python Package Index.
- The PyPI launch is **planned for late April, 2026**.

# Questions

Scan to download these slides:



<https://cburdine.github.io/files/lanlintroslides.pdf>

For additional questions and feedback, email: [colin\\_burdine1@baylor.edu](mailto:colin_burdine1@baylor.edu)

# Acknowledgments

- **Dr. Julia Chan's Lab** (Baylor University)
  - Superconductor Synthesis
- **Nora Bauer and Dr. George Siopsis** (University of Tennessee Knoxville)
  - Collaborators, Access to Quantinuum Hardware at ORNL
- IonQ and Microsoft Azure Quantum
  - Access to Quantum Hardware, Compute Credits
- **Nischal Gautam** (Baylor University)
  - Collaboration on NEGF extensions of PyQuINN.
- **Dr. Enrique P. Blair** (Baylor University)
  - Advisor, Mentor, Collaborator

# First-Author Publications

- [BBSB25] **C. Burdine**, N. Bauer , G. Siopsis , E. P. Blair. “Efficient Simulation of Open Quantum Systems on NISQ Trapped-Ion Hardware”. In: *Advanced Quantum Technologies* (2025), p. 2400606
- [BB25] **C. Burdine**, E. P. Blair. “Trotterless simulation of open quantum systems for NISQ quantum devices”. In: *Advanced Quantum Technologies* 8.1 (2025), p. 2400240
- [BB23] **C. Burdine**, E. P. Blair. “Discovery of novel superconducting materials with deep learning”. In: 2023 IEEE International Conference on Quantum Computing and Engineering (QCE). vol. 1. IEEE. 2023, pp. 1335–1341
- [BB26] **C. Burdine**, E. P. Blair. “Quantum-Informed E(3)-Equivariant Neural Networks for Material Design and Quantum Device Engineering”. In: (*In Preparation*)

## Presentations and Talks

- [Bur25b] **C. Burdine**. “Quantum-Informed Machine Learning for Rapid Material Design and Quantum Device Engineering”. In: Texas Quantum Summit (College Station, TX, Sept. 19–21 2025). 2025
- [Bur25a] **C. Burdine**. “Materials + ML”. in: 2025 Baylor Materials and Machine Learning Workshop (Waco, TX, June 9–20 2025). 2025
- [Bur24] **C. Burdine**. “Efficient Simulation of Open Quantum Systems on NISQ Quantum Computers”. In: Baylor University ECS Research Showcase (Waco, TX, Apr. 26, 2024). 2024
- [Bur23a] **C. Burdine**. “Discovery of novel superconducting materials with deep learning”. In: IEEE International Conference on Quantum Computing and Engineering (QCE) (Bellevue, WA, Sept. 16–21, 2023). 2023
- [Bur23b] **C. Burdine**. “Predicting the Critical Temperature of Doped and Alloyed Superconductors”. In: Southwest Data Science Conference (Waco, TX, Mar. 24, 2023). 2023

# Thank you!

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## **Additional Slides and References**

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## **ALIGNN Model Details**

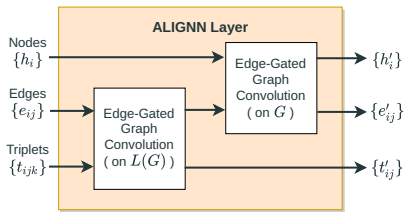
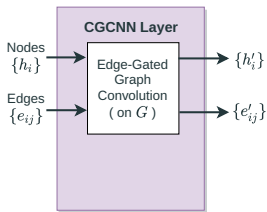
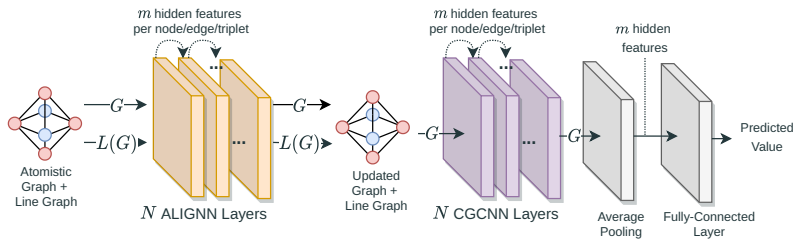
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# ALIGNN Model Summary

- I used the **Atomistic Line Graph Neural Network (ALIGNN)** model for both classifying superconductors and predicting  $T_c$  [CD21].
- ALIGNN is one of the top performing deep learning models for structure-based material property predictions. [DWG<sup>+</sup>20].
- Pros:
  - Is naturally invariant under  $E(3)$  (Euclidean) symmetries and space group symmetries.
  - Time complexity is  $\mathcal{O}(n)$  (by comparison DFT is  $\mathcal{O}(n^3)$ ).
  - Incorporates both bond lengths and bond angles.
- Cons:
  - Bond features are **not quantum-informed**; Lacks interpretability.
  - Prone to overfitting and **requires lots of data to perform well**.
  - Cannot distinguish the chirality of structures (not fully **equivariant**).

# ALIGNN Model

## ALIGNN Model Architecture [XG18][CD21]



# Superconductor Model Evaluation

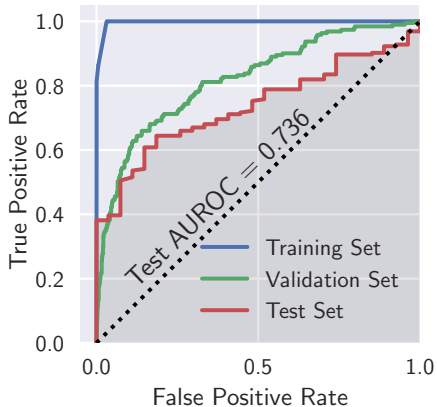
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# Model Evaluation

## Classifying Superconductors (Confusion Matrix and ROC Curve)

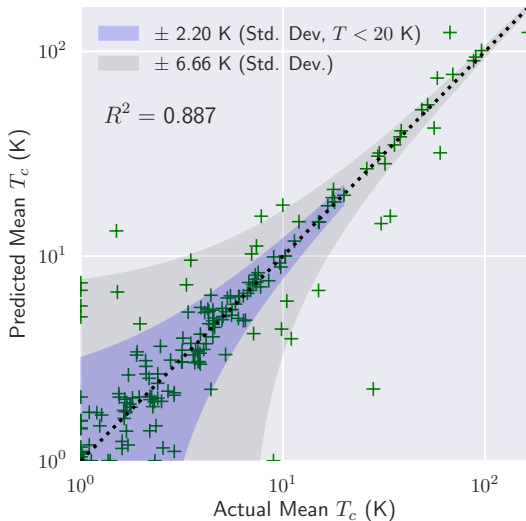
|                      |     | Predicted Superconductor  |   |
|----------------------|-----|---|---|
|                      |     | Yes   | No  |
| Contained in Dataset | Yes | <b>Total: 102</b><br>TaSe <sub>3</sub> , AlCu,<br>W <sub>2</sub> C, Nb <sub>3</sub> ,<br>RhGe, Ce,<br>TiNiSe <sub>2</sub> , Srlr <sub>2</sub> | <b>Total: 92</b><br>LaPd <sub>2</sub> Al <sub>3</sub> ,<br>Ru <sub>2</sub> Zr, In <sub>3</sub> Sn,<br>La <sub>3</sub> Sn, PdZr <sub>2</sub>                 |
|                      | No  | <b>Total: 95</b><br>YbLuB <sub>24</sub> ,<br>Ti <sub>3</sub> Ir*, CePb <sub>3</sub> *,<br>SrAu <sub>5</sub> , SrGe <sub>3</sub> *             | <b>Total: 917</b><br>Sc <sub>3</sub> Nb, Yb <sub>5</sub> Sn <sub>3</sub><br>Ba(SmSe <sub>2</sub> ) <sub>2</sub> ,<br>YAsO <sub>4</sub> , Th <sub>2</sub> Zn |

\* Materials that are actual superconductors, but were not contained in the dataset.



# Model Evaluation

## Predicting Empirical $T_c$ Distributions

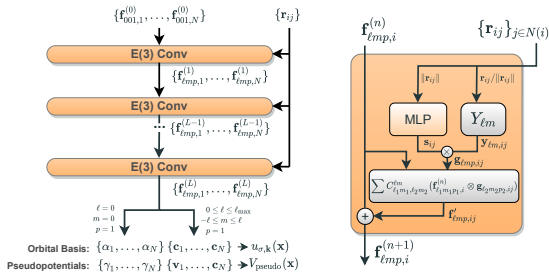


## More About QuINNs

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# E(3) Equivariant Graph Neural Networks

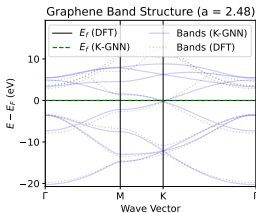
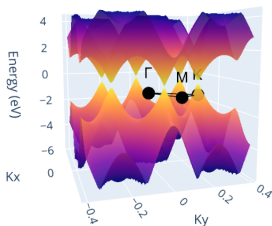
- To predict the orbital and pseudopotential parameters we employ **E(3)-equivariant graph neural network** layers.
- These layers respect all crystal translation, rotation, and reflection symmetries in **E(3)** (the Euclidean group) and parameterize the Gaussian orbital & potential coefficients  $\alpha_i$ ,  $c_i$  and  $\gamma_i$ ,  $v_i$ :



- $E(3)$  graph convolutions are performed in terms of spherical harmonic coefficients  $\mathbf{f}_{\ell mp}$  (called **irreducible representations**).

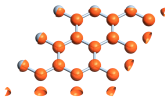
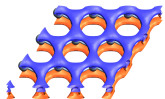
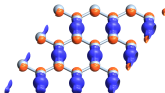
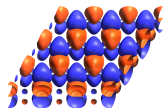
# Example: QuINN Carbon Allotropes Model

- We first trained a QuINN model on a small dataset of graphene atomic structures.
- Graphene is a 2D material consisting of C atoms bound in a honeycomb lattice.



- The model predicts the clear presence of Dirac cones at the  $\vec{k} = K$  points.
- The delocalized  $p_z$  orbitals forming these cones are responsible for graphene's high carrier mobility and exceptional conductivity properties.

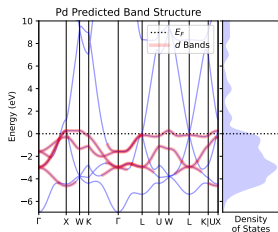
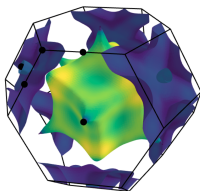
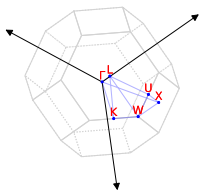
QuINN  $\Gamma$ -point  
Bloch Orbitals:



# Fermi Surfaces

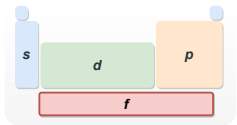
- An important feature of conducting or semiconducting materials is the topology of the bands that intersect the Fermi energy  $E_F$ .
- These band isosurfaces at  $E_F$  comprise the **Fermi Surface** and play an important role in the chemical and topological properties of materials.
- QuINNs allow for rapid high-resolution computation of Fermi surfaces, which is computationally expensive with DFT.

## Example: FCC Palladium Fermi Surface

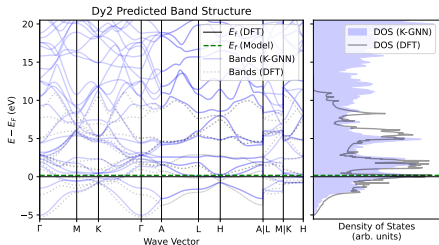
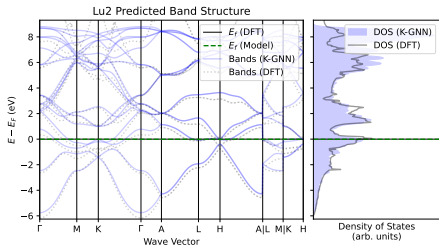


# Limitations of QuINNs

- QuINNs can predict valence band structure for all elements of the periodic table, attaining reasonable accuracy on  $f$ -block metals, like Lu and Dy.



- However, QuINNs have **limited capacity to capture conducting states** accurately for heavy atoms.

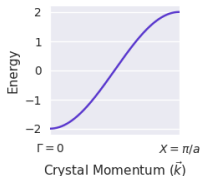
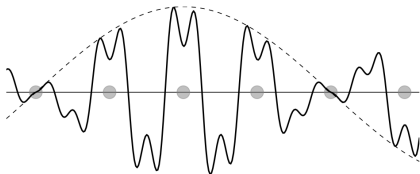


# Calculation of Tight-Binding Hamiltonians

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# Tight Binding Models

- According to **Bloch's Theorem**, a wave function  $\Psi(\vec{x})$  for a periodic crystal satisfies  $\Psi(\vec{x}) = e^{i\vec{k}\cdot\vec{r}}\Psi(\vec{x} - \vec{r})$ , where  $\vec{k}$  is a crystal momentum vector and  $\vec{r}$  is a translation vector.



- $\hat{H}$  can then be written as a periodic function of the momentum  $\vec{k}$ :

$$\hat{H}(\vec{k}) = \sum_{i,j \in (\text{cell})} e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)} \tilde{h}_{ij} \hat{c}_i^\dagger \hat{c}_j \quad (4)$$

- $H(\vec{k})$  has a continuum of allowed energies (the **band structure**)

# Representing Atomic Orbitals

- The effective Hamiltonian coefficients  $\tilde{h}_{ij}$  can be approximated as

$$\tilde{h}_{ij} = \langle \phi_i | H | \phi_j \rangle = \int \phi_{n_i}^*(\vec{x} - \vec{r}_i) H \phi_{n_j}(\vec{x} - \vec{r}_j) d\vec{x} \quad (5)$$

- The integral (5) cannot be evaluated in closed form for Hydrogen-like orbitals  $\phi_{n_i}(r, \theta, \varphi)$  and potentials  $V(r) \propto 1/r$ .
- We use orbitals  $\phi(r, \theta, \varphi) = F(r)X'_{\ell,c}(r, \theta, \varphi)$ , where  $X'_{\ell,c}$  are the (**real-valued**) solid harmonics:

$$X'_{\ell,c}(r, \theta, \varphi) = i^{n_c} \sqrt{\frac{2\pi}{2\ell+1}} r^\ell [Y_{\ell,m}(\theta, \varphi) \pm Y_{\ell,-m}(\theta, \varphi)] \quad (6)$$

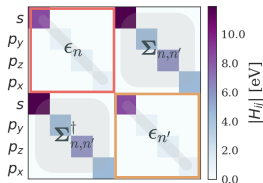
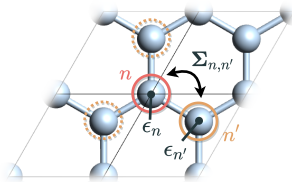
(Above, we use  $c = \ell + m$ , where  $\pm$  denotes  $\text{sign}(m)$  and  $n_c \in \{0, 1, 2, 3\}$ )

- We also expand  $F(r)$  and  $V(r)$  as sums of primitive Gaussians:

$$F(r), V(r) \propto \sum_{s=1}^{N_s} c_s e^{-\alpha_s r^2} \quad (7)$$

# Gaussian Orbitals and the Tight-Binding Hamiltonian

- We can expand the orbitals and potentials as sums of Gaussian functions times real solid harmonics (this allows the coefficients  $\tilde{h}_{ij}$  to be computed **in a closed differentiable form**).
- From the  $\tilde{h}_{ij}$  coefficients, we construct a block-matrix  $\mathbf{H}(\vec{k})$  of on-site energies  $\epsilon_n$  and neighboring site tunneling energies  $\Sigma_{n,n'}$ :







- The matrix  $\mathbf{H}(\vec{k})$  can be diagonalized for each  $\vec{k}$  to obtain eigenvalues (**band structure**) and eigenvectors (**Bloch orbitals**).





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